BEM modeling for ECT simulation of complex narrow cracks in multilayered structures

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• Problematic of narrow cracks detection in industrial applications

• Modeling of Eddy Current Testing (ECT): semi-analytical approaches studied
  – Volumetric Integral Method (VIM), limitation of the approach
  – Boundary Element Method (BEM)

• Calculation of dyadic Green Function (DGF) in multilayered structures

• BEM in multilayered structures with multiple cracks
  – Experimental validation
  – Numerical validation with Finite Element Method (FEM) code

• Coupled BEM-VIM approach for ECT simulation

• Conclusions and perspectives
Problematic of Narrow cracks detection

- Support of simulation tools to industrial applications of NDT
  - Probe design, Understanding of experimental signals, NDT procedures setup, Demonstration of device performance
  - ECT simulation of non ferromagnetic planar multilayered structures

- Characteristics of narrow cracks:
  1. Very small opening size compared to other flaw dimensions
  2. Complex cracks profiles
  3. Presence of electrical contacts between the crack faces (conductivity bridges)

**Issue hard to address with classical methods in a rapid way**

Examples:
- Fatigue cracks in structures under mechanical stress
- Stress corrosion cracking (SCC)
ECT modeling of narrow cracks with BEM

- Assumptions of the BEM approach:
  1. No current passing through the crack
  2. Crack opening $\leq \delta/2$ [Theo10] and $< \phi_{\text{coil}}$
  3. Main crack faces are parallel

1. Computation primary field $\mathbf{E}_p(\mathbf{r}) \quad r \in S_0, \Omega$

2.a Surface (ideal) crack case (Ass. 1)

$$\mathbf{n} \cdot \mathbf{E}_p(\mathbf{r}) = \lim_{\text{opening} \to 0} -i\omega\mu_0 \int_{S_0} G^{nn}(\mathbf{r}, \mathbf{r}') p(\mathbf{r}') \, dS_0$$

2.b Narrow crack case (Ass. 1, 2, 3)

$$\mathbf{n} \cdot \mathbf{E}_p(\mathbf{r}) = -i\omega\mu_0 \int_{\Omega} G^{nn}(\mathbf{r}, \mathbf{r}') p(\mathbf{r}') \, d\mathbf{r}'$$

3. Coil response

$$\Delta V = \frac{1}{I} \int_{\Omega \text{ or } S_0} p(\mathbf{r}) \mathbf{n} \cdot \mathbf{E}_p(\mathbf{r}) \, d\mathbf{r}'$$

Objective: generalization to a multilayered plate containing multiple cracks

Spectral domain DGF in multilayered structure

- **DGF via vector wave function treatment** [ChXi10]

\[
G_{nm} (\mathbf{r}, \mathbf{r}') = k_m^2 (\nabla \times \hat{z}) (\nabla' \times \hat{z}) g^{TE} (\mathbf{r}, \mathbf{r}') + \frac{k_m^2}{k_{nm}^2} (\nabla \times \nabla \times \hat{z}) (\nabla' \times \nabla' \times \hat{z}) g^{TM} (\mathbf{r}, \mathbf{r}')
\]

\[
k_n^2 = i\omega\varepsilon_n\mu_0, \quad k_m^2 = i\omega\varepsilon_m\mu_0
\]

- **Sommerfeld integrals** [ChXi10] & [ChCh03]

\[
g^{TE} (\mathbf{r}, \mathbf{r}') = \frac{i}{4\pi} \int_0^\infty \frac{1}{k_{mz}k_\rho} \left[ F_{emn}^{TE} (k_\rho, z, z') J_0 (k_\rho \rho) \right] dk_\rho
\]

\[
g^{TM} (\mathbf{r}, \mathbf{r}') = \frac{i}{4\pi} \int_0^\infty \frac{1}{k_{mz}k_\rho} \left[ F_{emn}^{TM} (k_\rho, z, z') J_0 (k_\rho \rho) \right] dk_\rho
\]

\[
k_\rho^2 = k^2 - k_z^2
\]

BEM in multilayered structure: multiple cracks topology

**BEM state equation with multiple cracks**

\[ \mathbf{n} \cdot \mathbf{E}_{i}^{inc}(\mathbf{r}) = -i \omega \mu_{0} \sum_{j=1}^{S=2} \int_{\Omega_{j}} G_{ij}^{mn}(\mathbf{r}, \mathbf{r}') p_{j}(\mathbf{r}') \, d\mathbf{r}' \quad \forall i \in \{1, S = 2\} \]

All cracks openings are oriented toward direction \( \mathbf{n} \)

\[ \mathbf{J}_{i}^{inc}(\mathbf{r}) \cdot \mathbf{n} = -\sum_{j=1}^{S=2} f_{i}(\mathbf{r}) \kappa_{i}^{2} \int_{\Omega_{j}} G_{ij}^{mn}(\mathbf{r}, \mathbf{r}') p_{j}(\mathbf{r}') \, d\mathbf{r}' \quad \forall i \in \{1, S = 2\} \]

Contrast function \( f_{i}(\mathbf{r}) : f_{i}(\mathbf{r}) = [\sigma(\mathbf{r}) - \sigma_{i}] / \sigma_{i} \)

**System of 2 integral equations**

Eq. 1 written in frame \((x_{1}, y_{1}, z_{1})\):

\[ J_{1}^{inc}(\mathbf{r}) = J_{1}^{inc}(\mathbf{r}) \cdot \mathbf{n}_{1} = -i \omega \mu_{0} \sigma_{1} f_{1}(\mathbf{r}) \int_{V_{1}} G_{11}^{n_{1} n_{1}}(\mathbf{r}, \mathbf{r}') \cdot p_{1}(\mathbf{r}') \, d\mathbf{r}' - i \omega \mu_{0} \sigma_{1} f_{1}(\mathbf{r}) \int_{V_{2}} G_{12}^{n_{1} n_{2}}(\mathbf{r}, \mathbf{r}') \cdot p_{2}(\mathbf{r}') \, d\mathbf{r}' \]

Similarly, Eq. 2 written in frame \((x_{2}, y_{2}, z_{2})\)
BEM in multilayered structure: multiple cracks topology

- Linear system obtained after the application of the Method of Moments (MoM)

\[
\begin{bmatrix}
J_1^{inc} \\
J_2^{inc}
\end{bmatrix} = 
\begin{bmatrix}
G_{11}^{n_1n_1} G_{12}^{n_1n_2} \\
G_{12}^{n_2n_1} G_{22}^{n_2n_2}
\end{bmatrix}
\begin{bmatrix}
p_1 \\
p_2
\end{bmatrix}
\]

Matrix size: \((N_1 + N_2) \times (N_1 + N_2)\)

- Off-diagonal terms

\[
G_{12}^{n_1n_2} = G_{12}^{xx} (\mathbf{r}, \mathbf{r}') \cos \theta_{12} + G_{12}^{yx} (\mathbf{r}, \mathbf{r}') \sin \theta_{12}
\]

\[
G_{21}^{n_2n_1} = G_{21}^{xx} (\mathbf{r}, \mathbf{r}') \cos \theta_{21} + G_{21}^{yx} (\mathbf{r}, \mathbf{r}') \sin \theta_{21}
\]

- Impedance variation (reciprocity theorem)

\[
\Delta Z = \sum_{s=1}^{S=2} \frac{1}{I^2 \sigma_s} \int_{V_s} J_s^{inc} (\mathbf{r}) \mathbf{p}_s (\mathbf{r}) \, d\mathbf{r}, \quad \mathbf{r} \in V_s, \quad s \in 1, 2
\]

\(I\): driving current (A)
Experimental validation

- Test case with 45° angle between the two cracks

First layer thickness: 1 mm
Conductivity: 18.72 MS.m⁻¹

Second layer thickness: 0.008 mm
Conductivity: dielectric insulator

Third layer thickness: 1 mm
Conductivity: 17.4 MS.m⁻¹

Work frequency: 1.5 kHz

Crack 1 (l x w x d): (29.65 x 0.198 x 1) mm
Crack 2 (l x w x d): (29.89 x 0.234 x 2) mm

BEM unknowns:
\[ N_{x1} \times N_{y1} \times N_{z1} = 1 \times 60 \times 3 \]
\[ N_{x2} \times N_{y2} \times N_{z2} = 1 \times 60 \times 5 \]
Experimental validation

CPU time: 1m30s (complete map, 60 x 60 points)
Numerical Comparison with FEM results

- Inspection of a plate affected by a complex crack

- Crack modeled by a set of 5 notches
  - Common depth: 2 mm
  - Common opening: 0.2 mm
  - 5 different lengths: [4.75 5.2 2.5 5 4] mm
  - 5 different skew angles

- BEM discretization: less than 1500 elements (uniform mesh of the flaws only)
- FEM discretization: around 70 000 elements (3D mesh of the configuration)
Numerical Comparison with FEM code

- Multiple arbitrarily oriented cracks in a plate structure

Signal extraction

Complex plane extracted signal
Numerical comparison with FEM code

Signal extraction at -1.5 mm from the cartography centre

Signal extraction at the cartography centre

Signal extraction at 1.5 mm from the cartography centre

BEM computation time (6561 positions): ~13 min

FEM computation time (1 position): ~7 min
Coupled BEM-VIM approach

• Motivation:
  Efficient modeling approach for the simulation of configurations involving both volume flaws and narrow cracks

• Open issues:
  Narrow and very narrow cracks not easy to model with FEM and VIM (very large meshes required)
  Volume cracks like boreholes not treatable with BEM (model assumptions no more true)

• Proposed solution:
  Modeling in the same simulation of narrow cracks with BEM and of volume flaws with VIM
Coupled BEM-VIM approach

Comparison with a complete VIM modeling

Computation time for 60 x 60 points map:

BEM-VIM ~ 4 min.
VIM ~ 8.5 min.

Signal extraction

Experimental validations in progress
Conclusions

Development of BEM model for the ECT simulation of stratified planar media affected by complex narrow cracks

Efficient computation of the dyadic Green function
- Source and observation located in different layers
- Excellent agreement with the experiments
- Low CPU time due to 2D mesh on the crack

Experimental validations of the model

Interactions between multiple cracks (FEM comparisons)
- Very good agreement with FEM results
- Small computation time with respect to FEM

Coupled approach between BEM and VIM
Conclusions

Efficient simulation tools dedicated to realistic ECT configurations
• Modeling of tilted narrow cracks

• Cracks with more complex shapes using RWG basis and testing functions (more precise description than rectangular cells)

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