Modelling of Specimen Interaction with Ferrite Cored Coils by Coupling Semi-Analytical and Numerical Techniques

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OUTLINE

• Use of ferrite-cored probes in eddy-current testing (ECT) applications

• Problem splitting: motivation and coupling principle

• Axisymmetric probes: Finite integration technique (FIT) /semi-analytical coupling
  • FIT discretisation scheme
  • Internal problem solution using the FIT 2½ magnetostatic formulation
  • Propagation of the solution
  • Reflection treatment using the modal approach

• 3D probes: Surface integral equation (SIE) /semi-analytical coupling
  • Overview of the SIE approach

• Demonstration example: tilted axisymmetric ferrite-cored probe over a conducting half-space

• Conclusions and perspectives
PROBLEM SCOPE: FERRITE-CORED PROBES IN ECT APPLICATIONS

Ferrite-cored industrial probes for performance optimisation

- STT, STL (steam generator tube inspection, EDF France)
- +Point (steam generator tube inspection, US-nuclear park)
- Rototest, probes with C and E cores (fastener inspection, aeronautical applications)

Lift-off and tilt variations are a significant noise source to the measured eddy-current signal

Strong coupling between ferrite core and specimen
Non-symmetric (3D) arrangement of symmetrical objects

Problem decomposition into sub-problems of simpler geometry

(Uniqueness theorem)   (Equivalence theorem)
OVERVIEW OF THE FINITE INTEGRATION TECHNIQUE

Grid Maxwell’s equations:

\[
\begin{align*}
\hat{C}\hat{e} &= -j\omega \hat{b}, \\
\hat{C}\hat{h} &= j\omega \hat{d} + \hat{j}, \\
\hat{S}\hat{d} &= q, \\
\hat{S}\hat{b} &= 0.
\end{align*}
\]

Discrete constitutive equations:

\[
\begin{align*}
\hat{d} &= M_\varepsilon \hat{e}, \\
\hat{b} &= M_\mu \hat{h}, \\
\hat{j} &= M_\kappa \hat{e}.
\end{align*}
\]

Discrete grad operator:

\[
G = -\hat{S}^T
\]

- **Grid duality:**
  \[
  \hat{C} = C^T, \quad \hat{S} = S^T
  \]
- **Topological properties** of the discrete operators:
  \[
  SC = 0, \quad CG = 0
  \]

Yee-like dual system of orthogonal grids

Applicable in rectangular **cylindrical** and spherical grid system

Magnetostatic formulation for loss-less medium

Solution decomposition into a partial solution and the solution of the homogeneous problem:

\[
\mathbf{h} = \mathbf{h}_i - \mathbf{G}\phi
\]

with

\[
\mathbf{C}\mathbf{h}_i = \mathbf{j}_s
\]

\[
\mathbf{C}\mathbf{G}\phi = 0
\]

Axisymmetrical geometries: Modal decomposition along the direction of invariance and 2D meshing on the transversal plane

\[
\phi(\phi) = \sum_{m=0}^{\infty} \phi_m e^{im\phi}
\]

Discrete equation for the scalar potential

\[
\left[\tilde{S}\mathbf{M}_\mu \mathbf{G} - \alpha_m^2 \mathbf{M}_\mu^N\right] \phi_m = \tilde{S}\mathbf{M}_\mu \mathbf{h}_{i,m} + q_m \quad \text{for} \quad m = 0, \pm 1, \ldots \infty
\]

with \( \alpha_m = m\Delta\phi \)

Boundary terms
Non-symmetric (3D) arrangement of symmetrical objects

Problem decomposition into sub-problems of simpler geometry
Second Green’s theorem

\[ \mathbf{B}(\mathbf{r}) = -\nabla \int_{\partial \Omega} \left[ g(\mathbf{r}, \mathbf{r}') \frac{\partial \Phi(\mathbf{r}')}{\partial n'} - \frac{\partial g(\mathbf{r}, \mathbf{r}')}{\partial n'} \Phi(\mathbf{r}') \right] \, dS' \]

with \( g(\mathbf{r}, \mathbf{r}') \) the scalar Green’s function of the free space

Decomposition of the Green’s function in cylindrical functions

\[ g(\mathbf{r}, \mathbf{r}') = \sum_{m=-\infty}^{\infty} e^{im(\varphi - \varphi')} \int \frac{g_o^m(\rho, z, \kappa)g_s^m(\rho', z', \kappa)}{\kappa} \, d\kappa \]

with \( g_s^m(\rho', z', \kappa) = \left\{ \begin{array}{ll} J_m(\kappa \rho') e^{\pm \kappa z'} & \text{if } m \neq 0 \\ \frac{1}{\pi} I_m(\kappa \rho') e^{i \kappa z'} & \text{if } m = 0 \end{array} \right. \) \( g_o^m(\rho, z, \kappa) = \frac{\mu_0}{4\pi} \left\{ \begin{array}{ll} J_m(\kappa \rho) e^{\pm \kappa z} & \text{if } m \neq 0 \\ K_m(\kappa \rho) e^{i \kappa z} & \text{if } m = 0 \end{array} \right. \)

Application to field integral representation

\[ \mathbf{B}(\mathbf{r}) = -\sum_{m=-\infty}^{\infty} \int G_s(\kappa) \nabla g_o^m(\rho, z, \kappa) \, d\kappa \]

with \( G_s(\kappa) = 2\pi \sum_{i \in S} \left[ g_s^m(\rho_i, z_i, \kappa) b_{i,m} - \frac{\partial g_s^m(\rho_i, z_i, \kappa)}{\partial n_i} \phi_{i,m}(\rho_i, z_i) \right] \)

- No need of special treatment for the Green’s function singularity
- Memory efficient
Non-symmetric (3D) arrangement of **symmetrical objects**

Problem decomposition into sub-problems of simpler geometry
2D Fourier transform of the normal incident magnetic field

\[
\widetilde{B}_{z}^{inc}(u, v) = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} B_z(x, y, z = 0) e^{-iux} e^{-ivy} dx dy
\]

Reflection coefficient in the Fourier Domain (half-space)

\[
R(u, v) = \frac{a \mu_r - a_1}{a \mu_r + a_1}
\]

Calculation of the reflected field via inverse Fourier transform

\[
B^{ref}(r) = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\widetilde{B}_{z}^{inc}(u, v)}{a} R(u, v) e^{-az} (iu e_x + iv e_y - a e_z) e^{iux} e^{ivy} dudv
\]

with \( \alpha = \sqrt{u^2 + v^2} \)

- Straight-forward generalisation for multi-layered work-pieces
- Applicable to other canonical geometries (tubes, etc.)

Non-symmetric (3D) arrangement of symmetrical objects

Problem decomposition into sub-problems of simpler geometry
Linear isotropic piecewise homogeneous permeable media with boundary $\Gamma$

The magnetic field is given by
\[
H(\mathbf{r}) = H_{\text{inc}}(\mathbf{r}) - \frac{1}{4\pi} \int_{\Gamma} \sigma(\mathbf{r}') \nabla \frac{1}{||\mathbf{r} - \mathbf{r}'||} \, d\mathbf{r}' \quad \forall \mathbf{r} \in \mathbb{R}^3 \setminus \Gamma
\]

where $\sigma = \mathbf{M} \cdot \mathbf{n}$ is the unknown of the (reduced scalar potential) integral formulation
\[
\frac{1}{2} \sigma(\mathbf{r}) + \left( \frac{\mu_r^\text{int} - \mu_r^\text{ext}}{\mu_r^\text{int} + \mu_r^\text{ext}} \right) \frac{1}{4\pi} \int_{\Gamma} \sigma(\mathbf{r}') \nabla \frac{1}{||\mathbf{r} - \mathbf{r}'||} \cdot \mathbf{n}(\mathbf{r}) \, d\mathbf{r}' = \left( \frac{\mu_r^\text{int} - \mu_r^\text{ext}}{\mu_r^\text{int} + \mu_r^\text{ext}} \right) H_{\text{inc}}(\mathbf{r}) \cdot \mathbf{n}(\mathbf{r}) \quad \forall \mathbf{r} \in \Gamma.
\]

**High-order polynomial expansion** for both the geometry and the unknown

- The number of unknowns is reduced for a given accuracy
- It allows the use of a direct solver
- A single LU decomposition is performed for all positions and iterations
Tilted axisymetrical ferrite-cored probe over conducting half-space

\[ \sigma = 18 \text{ MS/m} \]

<table>
<thead>
<tr>
<th>Coil parameters</th>
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<tbody>
<tr>
<td>Inner radius ( r_{in} )</td>
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<tr>
<td>Outer radius ( r_{out} )</td>
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<tr>
<td>Length ( l )</td>
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<tr>
<td>Number of turns ( N )</td>
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<th>Core parameters</th>
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<tbody>
<tr>
<td>Radius ( r )</td>
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<tr>
<td>Length ( l )</td>
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<tr>
<td>Relative permeability</td>
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Frequency \( f = 1 \) KHz
Normal magnetic field on the interface of the half-space

Tilt angle $\theta = 20^\circ$

FEM mesh

SIE mesh

- **FEM (COMSOL)**
- **FIT/Semi-Analytical**
- **SIE/Semi-Analytical**

**FEM:** 10 min (600,000 elements, PC 24 GB, Xeon 8 cores)

**FIT/Semi-Analytical:** ~5 min

**SIE/Semi-Analytical:** ~5 min

(standard PC)
RESULTS: CONVERGENCE STUDY

FIT/Semi-Analytical Coupling:

\[ \frac{|W_m^{(i)} - W_m^{(i-1)}|}{|W_m^{(i)}|} \]

Convergence criterion: \[ W_m^{(i)} \] : Magnetic energy at \( i \)th iteration

SIE/Semi-Analytical Coupling:

\[ \frac{|X^{(i)} - X^{(i-1)}|}{|X^{(i)}|} \]

Convergence criterion: \[ X^{(i)} \] : Internal problem solution at \( i \)th iteration
Coil impedance as a function of the tilt angle

\[ R(\theta) \]

\[ X(\theta) \]

\[ \frac{|Z_{FEM} - Z_{Coupl.}|}{|Z_{FEM}|} \leq 0.8\% \]

\[ \left| \arg(Z_{FEM}) - \arg(Z_{Coupl}) \right| \leq 0.4 \text{ deg.} \]
Ferrite-cored vs. air-cored coil impedance

Results for the air-cored coil computed using (Theodoulidis 2005)
Normal component of the induced magnetic field on the half-space interface
CONCLUSIONS AND PERSPECTIVES

• Combination of semi-analytical and numerical approach for the modelling of ferrite cored probes over planar specimen
• Reduced computational time in respect to 3D meshing of the entire problem (~x3 speed-up)
• No limitations concerning the lift-off
• Reduced computational noise
• Automatic 2D grid, lift-off and tilt parameterisation

Work under way

• Coupling with semi-analytical solutions for tubes and fastener structures

Compatible with flaw response models of CIVA

• Coupling with 2 ½D FIT approach for taking into account pieces with irregular profile